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# Binding of holes induced by gauge fields

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**Abstract.** We show in the two-band Hamiltonian for CuO layers that holes bind due to gauge fields induced by antiferromagnetic fluctuations. Superconductivity appears as a superfluid transition of pairs of holes.

## 1. Introduction

The purpose of this paper is to show that superconductivity in high-$T_c$ layer compounds is due to gauge fields induced by antiferromagnetic (AF) fluctuations in the disordered phase. These gauge fields produce an attractive interaction leading to binding of holes. As a result, superconductivity (SC) appears as a superfluid transition of pairs of holes with a charge of $2e$.

Explicitly we assume the following Hamiltonian for the CuO planes. The Cu $3d_{x^2-y^2}$ bonding band remains half-filled when we dope the system. The holes go into the oxygen band $O^-$ of type $p_\sigma$. This leads to a square lattice of spin-1/2 AF Heisenberg interaction with a coupling constant $J$ between the copper spins in the plane and a Kondo coupling $I$ between copper and oxygen. This model has been used in previous works [1–8]. In addition we assume a small antiferromagnetic coupling $J_x = J_\eta$ between copper–copper in the $z$ direction perpendicular to the CuO layers. We also assume that the Kondo coupling $I$ is larger than $J$ and the hopping constant $t$ ($t = 1/m$) is smaller than $I$.

The binding of holes emerges in a special case when the quantum Heisenberg antiferromagnet is described by the quantum disordered phase. The quantum disordered phase is established for the non-linear sigma model in $d + 1$ dimensions for coupling constants $J < J_c$ ($J_c$ is the critical coupling for the Néel state). The existence of a critical coupling $J_c$ for the quantum spin-1/2 Heisenberg antiferromagnet is established for a frustrated Heisenberg model (the presence of holes induces next-nearest-neighbour coupling).

In this paper we represent the Heisenberg antiferromagnet by a non-linear sigma model in $d + 1$ dimensions. The crucial assumption is that $J < J_c$; therefore the quantum Heisenberg antiferromagnet is described by the quantum disordered phase. In the disordered phase the following new picture arises. The copper spin variables are described by two charged bosons (the $z_1$, $z_2$ variables used in the $CP^1$ representation). The excitation for the bosons (the $z_1$, $z_2$ variables) has a finite gap. This picture ceases to be correct when one approaches the Néel state ($J = J_c$): the gap vanishes and one of the

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bosons condenses, leaving one gapless boson, which appears as a spin-wave excitation. In the quantum disordered phases the massive charged bosons are coupled to a massless gauge field. The fluctuations of the gauge field are identical to the electromagnetic fluctuations. When the system is doped the charged holes attract each other due to the exchange of the gauge field, similar to the attraction of opposite electric charges in electrodynamics, which exchange a photon.

This paper is organized in the following way. In section 2 we present the microscopic model, which contains the Heisenberg Hamiltonian, the kinetic energy of the holes and the coupling between the magnetic moments of the copper and the holes. Performing a gauge transformation we show that the coupling between the magnetic moments of the copper and the holes. Performing a gauge transformation we show that the coupling between the magnetic moments and the holes is replaced by an AF background field and a gauge field in which the holes are moving. It is the AF background field that gives rise to new opposite charges for the holes. (The charges are defined with respect to the gauge field determined by the fluctuations of the quantum disordered AF. Holes with spin up have an opposite sign to holes with spin down they couple to the gauge field.) Section 3 is devoted to the derivation of the hole–hole interaction. This calculation is performed using the following steps. The coupling between the magnetic moments of the copper and the holes is replaced by a staggered antiferromagnetic field (which acts as a potential for the holes) and an additional gauge field. The fluctuations of the gauge field are controlled by the AF Hamiltonian of the copper atoms. The gauge fields are described within the formalism of the disordered phase, which consists of two massive bosons coupled to a massless gauge field. The exchange of the gauge field between the massive bosons gives rise to an attraction between the holes. Section 4 is devoted to the binding problem and general discussions concerning the 2e superfluid.

## 2. The microscopic model

The action for this system is given by

$$S = S_0 + S_J + S_I \tag{1}$$

with

$$S_0 = \int_0^\beta \mathrm{d}\tau \sum_{\alpha=\uparrow,\downarrow} \int \mathrm{d}^2 r \{P_\alpha^+(r,\tau)[\partial_\tau + \mathcal{E}(\nabla^2)]P_\alpha(r,\tau)\} \tag{2}$$

$$S_J = \int_0^\beta \mathrm{d}\tau \left(\sum_n \frac{\Delta\tau}{8}[\partial_\tau \boldsymbol{\Omega}(n,\tau)]^2 + \frac{J}{2}\sum_{n,n'} \boldsymbol{\Omega}(n,\tau)\cdot\boldsymbol{\Omega}(n',\tau)\right.$$
$$\left. + \frac{J\eta}{2}\sum_{n_z,n_{z'}} \boldsymbol{\Omega}(n_z,\tau)\cdot\boldsymbol{\Omega}(n_{z'},\tau)\right) - \frac{\mathrm{i}}{2}\sum_n \Gamma_{\mathrm{WZ}}[\boldsymbol{\Omega}(n)] \tag{3}$$

and

$$S_I = \int_0^\beta \mathrm{d}\tau \left(I\sum_n \boldsymbol{\Omega}(n,\tau)P_\alpha^+(n,\tau)\cdot\boldsymbol{\sigma}_{\alpha\beta}0_\beta(n,\tau)\right). \tag{4}$$

Here $\mathbf{\Omega}(n)$ represents the spin-1/2 variable of $Cu^{2+}$ on a square lattice in the coherent-state [9, 10] representation ($\mathbf{\Omega} = (\Omega_1, \Omega_2, \Omega_3)$, $|\mathbf{\Omega}| = 1$). The p holes at the copper site have $d_{x^2-y^2}$ symmetry,

$$P_\alpha^+(n, \tau) = \sum_K f(K)\, e^{-iKn} P_\alpha^+(K, \tau)$$

with the function

$$f(K) = [1 - \tfrac{1}{2}(\cos K_x + \cos K_y)]^{1/2}.$$

$\hat{\varepsilon}(\nabla^2)$ is the kinetic energy operator for the holes with a large effective mass $m$. The kinetic energy of the holes in the $z$ direction is neglected and in the plane $\varepsilon(\nabla_\perp^2) \sim K^2/2m$.

$\Gamma_{WZ}$ is the Wess–Zumino (WZ) term, which incorporates the quantum-mechanical efects [9, 10]. At the classical level the action is invariant under the Mattis transformation $J \to -J$, $\mathbf{\Omega}(n) \to (-1)^n \mathbf{\Omega}(n) = \boldsymbol{M}(n)$. The Wess–Zumino term $\Gamma_{WZ}[\mathbf{\Omega}(n)]$ is not invariant under the Mattis transformation. Therefore the quantum Heisenberg AF is not invariant under this transformation. Recently it was shown that the sum of the WZ term vanishes for a square lattice [9, 10] and therefore the quantum Heisenberg AF in $d > 1$ dimensions is equivalent to the non-linear $\sigma$ model in $D = d + 1$ dimensions [11]. Working in the $CP^1$ representation ($\boldsymbol{M} = Z^+ \boldsymbol{\sigma} Z$, $Z^+ = (z_1^+, z_2^+)$, $|z_1|^2 + |z_2|^2 = 1$) one maps the Heisenberg AF to a two-component field $Z$ with a gauge field $\boldsymbol{A}$, which in the disordered phase becomes equivalent to the electromagnetic gauge field. We introduce a matrix $g \in SU(2)$, which satisfies the relation $\boldsymbol{M} \cdot \boldsymbol{\sigma} = g^{-1}\sigma_3 g$. Solving for the matrix $g$ we find [12]: $g_{11} = Z_1, g_{12} = Z_2, g_{21} = -Z_2, g_{22} = Z_1$. In the SU(2) representation the exchange Cu–O term takes the form:

$$\sum_n (-1)^n P^+(n, \tau)[\boldsymbol{M}(n, \tau) \cdot \boldsymbol{\sigma}] P(n, \tau)$$

$$= \sum_n (-1)^n P^+(n, \tau)[g^{-1}(n, \tau)\sigma_3 g(n, \tau)] P(n, \tau)$$

$$A_\mu = A_\mu^{EM} + (\partial_\mu g) g^{-1} \qquad \mu = (\boldsymbol{r}, \tau).$$

With the convention that $P(n, \tau)$ is a two-component spinor, we perform a local gauge transformation:

$$\psi^+(n, \tau) = P^+(n, \tau) g^{-1}(n, \tau)$$

$$\psi(n, \tau) = g(n, \tau) P(n, \tau)$$

$$A_\mu = A_\mu^{EM} + (\partial_\mu g) g^{-1}.$$

$A^{EM}$ represents the normal electromagnetic vector potential, which couples to the holes with a charge of $1e$. The gauge transformation separates the AF interaction into two parts: a constant antiferromagnetic background field and a gauge field $(\partial_\mu g) g^{-1}$. The constant antiferromagnetic background field will transform the two-component spinor $P(n, \tau)$ into a one-component spinless fermion with the spin determined by the position in space (odd sites spin up and even sites spin down or vice versa). This result is similar to the one derived in [12] (see equations (10) and (11) there) for the Hubbard model using the method of geometrical quantization.

In the $CP^1$ representation the Heisenberg AF action is decoupled with the aid of the gauge field $A_\mu = (-i/2)(Z^+ \partial_\mu Z - (\partial_\mu Z^+) Z)$ and the Lagrange multiplier $\lambda$ for the constraint $Z^+Z = 1$ (see [13] pp 139–42). The action given in equation (1) takes, in the continuum representation, the form

$$S = \int_0^\beta \mathrm{d}\tau \int \mathrm{d}^2 r \{\psi^+(r,\tau)[\partial_\tau + (\partial_\tau g)g^{-1}]\psi(r,\tau)$$
$$+ \psi^+(r,\tau)\hat{\varepsilon}([\partial_r + eA_\mu^{\mathrm{EM}} + (\partial_r g)g^{-1}]^2)\psi(r,\tau)$$
$$+ \tfrac{1}{2}I\cos(Qr)\psi^+(r,\tau)\sigma_3\psi(r,\tau)\} + \int_0^{\beta\hbar c} \mathrm{d}\tau \int \mathrm{d}^d r$$
$$\times \left(\frac{1}{2\gamma}(\partial_\mu + \mathrm{i}A_\mu)Z^+(\partial_\mu - \mathrm{i}A_\mu)Z + \lambda(r,\tau)[Z^+(t,\tau)Z(r,\tau) - 1]\right). \tag{5}$$

The relation between the gauge field and the $g \in \mathrm{SU}(2)$ matrix is given by

$$A_\mu = -[(\partial_\mu g)g^{-1}]_{1,1} = [(\partial_\mu g)g^{-1}]_{2,2}.$$

The wavevector $Q = (\pi/\alpha,\ \pi/a)$ reproduces the antiferromagnetic modulation $(-1)^n$; $\gamma$ is proportional to $J$, $\gamma = \gamma_0\Lambda^{2-(d+1)}$ ($a$ is the lattice constant, $\Lambda$ is a cutoff, $\Lambda \sim 1/a$, and $d$ is the spatial dimension), $\gamma_0 = (J\eta^{1/2}\hbar c)^{-1}$ ($c$ is the spin-wave velocity and $\eta = J_z/J$). For short wavelengths the coupling $J_z$ in the $z$ direction is negligible, leading to $\gamma_0 = (J\hbar c)^{-1}$. Similarly, at high temperature such that $\hbar c/T < \xi$ ($\xi$ is the magnetic correlation length), we neglect the temporal effects $\gamma = \gamma_0\Lambda^{2-d}$ with $\gamma_0 = (JT)^{-1}$, reducing the problem to the classical Heisenberg AF action.

In order to investigate the effect of the antiferromagnetic field on the holes, we consider first the effect of the constant antiferromagnetic field. We diagonalized the oxygen holes Hamiltonian in the absence of the gauge field. We introduce

$$\psi(r) = \sum_K f(K)C(K)\exp(\mathrm{i}Kr)$$

and the Hamiltonian for the holes takes the form

$$H = \sum_{K,\sigma=\uparrow,\downarrow} \varepsilon(K)C_\sigma^+(K)c_\sigma(K) + \tfrac{1}{2}I \sum_{K,\sigma=\uparrow,\downarrow} f(K)C_\sigma^+(K)[f(K+Q)C_\sigma(K+Q)$$
$$+ f(K-Q)C_\sigma(K-Q)]. \tag{6}$$

Here $\varepsilon(K)$ represents the energy bands for the holes and $I$ is the exchange coupling. Such a Hamiltonian was considered by a number of authors in the past [14]. In our case $Q > 2K_\mathrm{F}$ with $K_\mathrm{F}$ determined by the hole concentration $\delta$, $K_\mathrm{F} = \Lambda(2\pi\delta)^{1/2}$ in the two-dimensional planes. For $Q = 2K_\mathrm{F}$ equation (6) can be diagonalized exactly.

For $Q \neq 2K_\mathrm{F}$ we have to diagonalize an infinite matrix. This is performed by a sequence of $2 \times 2$ rotations in the space $K, K \pm Q, K \pm 2Q, \ldots$. Higher states such as $|K \pm 2Q\rangle$ can be neglected since the energies of the holes in the vicinity of the Fermi sphere satisfy $I/|\varepsilon(K) = \varepsilon(K \pm 2Q)| < 1$. The leading contribution comes from the states $|K\rangle$, $|K+Q\rangle$ and $|K\rangle$, $|K-Q\rangle$. In this subspace we diagonalize equation (6). We introduce eigenvector operators $\gamma_\sigma(K)$ and eigenvalues $\bar{\varepsilon}(K)$. We represent the original operators $C(K)$ in terms of the new ones and obtain

$$C_\sigma(K) = \gamma_\sigma(K)\cos\alpha_+(K)\cos\alpha_-(K)$$
$$- \sigma[\gamma_\sigma(K+Q)\sin\alpha_+(K) + \gamma_\sigma(K-Q)\sin\alpha_-(K)] \tag{7}$$

where

$$I(K, K \pm Q) = I^2 f(K) f(K \pm Q) \tag{8}$$

$$\cos 2\alpha_{\pm}(K) = \{1 + I(K, K \pm Q)/[\varepsilon(K) - \varepsilon(K \pm Q)]^2\}^{-1/2}. \tag{9}$$

In real space we define new spinors

$$\tilde{\psi}(r) = \sum_K \gamma(K) \exp(\mathrm{i}Kr).$$

Using this representation we rewrite the action given in equation (5). Averaging over the Fermi sphere we obtain that the gauge field couples only to spin up or spin down, and the mixing term vanishes [14]:

$$m^{-1}\psi^+(r,\tau)[(\partial_\mu g)g^{-1}]\psi(r,\tau) \simeq m^{*-1}\psi^+(r,\tau)[\mathrm{i}\sigma_3 A_\tau]\tilde{\psi}(r,\tau) \tag{10}$$

$$M^{*-1} = m^{-1}\langle\cos^2\alpha_+(K)\cos^2\alpha_-(K)\rangle_{\mathrm{FS}}. \tag{11}$$

The meaning of equation (10) is that holes with spin up couple to the gauge field $A_\tau$ and holes with spin down to the same gauge field with opposite sign $-A_\tau$. Since the gauge field $A_\tau$ is analogous to the scalar potential in electrodynamics, we may say that the holes have opposite charges (the $A_r$ component is equivalent to the ordinary vector potential in electrodynamics).

This result is consistent with the picture given in [12], where the spinor represents spin up at even sites and spin down at odd sites or vice versa. We mention that the validity of these results depends on the hole concentration. The hole concentration determines the Fermi energy. The mixing between states $|K\rangle$, $|K \pm Q\rangle$ is controlled by the ratio $t/I$ and the hole concentration $\delta$ ($K_{\mathrm{F}} = \Lambda(2\pi\delta)^{1/2}$). For $Q \simeq 2K_{\mathrm{F}}$ the mixing is strong and equation (10) is independent of $t/I$. For small hole concentration ($Q \gg 2K_{\mathrm{F}}$) equation (10) depends on $t/I$. Therefore, for a fixed value of $t/I$, equation (10) is valid for $\delta > \delta_{\min}$ with $\delta_{\min} \sim (t/I)^2$.

For the remaining part of the paper we consider hole concentrations such that $\delta > \delta_{\min}$ and use equations (10) and (11). The approximation involved in equation (10) replaces $\tilde{\psi}^+\sigma_3\tilde{\psi}$ by $\tilde{\psi}^+[\sigma_3 + \alpha(\sigma - \sigma_3)]\psi$, where $|\alpha| \ll 1$, $\langle\alpha\rangle = 0$ is the $xy$ anisotropy parameter representing the spin-flip contribution. Since $\alpha$ is small, the leading contribution will come from the 'Ising' part $\sigma_3$. Investigating the Heisenberg AF action we find that after integrating the '$Z$' variables the effective action depends only on two parameters, the gauge field $A$ and the Lagrange field $\lambda$ [13].

## 3. *The derivation of the effective hole–hole interaction*

The action that contains the holes and the AF Heisenberg Hamiltonian takes the form

$$S = \int_0^\beta \mathrm{d}\tau \int \mathrm{d}^3 r \Big[\tilde{\psi}^+(r,\tau)\Big((\partial_\tau + \mathrm{i}A_\tau\sigma_3) + \frac{1}{2m^*}(\partial_r + eA_r^{\mathrm{EM}} + \mathrm{i}A_r\sigma_3)^2\Big)\tilde{\psi}(r,\tau) + \Gamma(A_\mu,\lambda)\Big] \tag{12}$$

$$\Gamma(A_\mu,\lambda) = \int_0^\beta \mathrm{d}\tau \int \mathrm{d}^d r \left(NT_r L_n[-(\partial_\mu + \mathrm{i}A_\mu) + \lambda] - \frac{\lambda}{2\gamma}\right) \tag{13}$$

where $N = 2$.

We observe that the action given in equation (12) has two gauge fields $A^{\mathrm{EM}}$ and $A$. The first is the normal electromagnetic field with the positive charge $e$ (the electromagnetic charge of the holes). The second gauge field has the origin from the antiferromagnetic interaction. This gauge field couples the two charged bosons $z_1, z_2$. Owing to the Kondo coupling between the holes and the copper spins $\boldsymbol{M} = z^+\boldsymbol{\sigma} z$, the holes behave as if they had additional 'charges' (holes with spin up couple to $A_\tau$ and holes with spin down to $-A_\tau$). The absolute value of the 'charge' is obtained from the coupling constant, which governs the gauge fluctuations.

The action $\Gamma(A, \lambda)$ contains two parts: $\Gamma(0, \langle\lambda\rangle) + [\Gamma(A, \lambda) - \Gamma(0, \langle\lambda\rangle)]$. The first one determines the values $\langle\lambda\rangle = M^2$ with $M^{-1}$ being the correlation length of the two charged bosons $z_1, z_2$ and therefore the correlation length of the spin–spin correlations,

$$\langle \boldsymbol{M}(x) \cdot \boldsymbol{M}(y) \rangle = \langle z^+(x)\boldsymbol{\sigma} z(x) z^+(y)\boldsymbol{\sigma} z(y) \rangle \sim \sum_{\alpha,\beta} \langle z^+_\alpha(x) z_\alpha(y) \rangle \langle z^+_\beta(y) z_\beta(x) \rangle$$
$$\sim \exp(-2M|x-y|).$$

The second part $\Gamma(A, \lambda)$ will be shown to be analogous to the action of the elecromagnetic field. The strength of those fluctuations is controlled by the coupling constant $|M|$, which is also the inverse of the correlation length of the charged bosons $z_1, z_2$.

We expand the action $\Gamma(A, \lambda)$ in the large-$N$ expansion. Following the derivation given in [13] (pp 139–42) we have

$$\Gamma(A_\mu, \lambda = \langle\lambda\rangle + v) = \Gamma(0, \langle\lambda\rangle = M^2) + \frac{N}{2} \sum_q [v(q)\pi(q)v(-q) + A_\mu(q)\pi_{\mu\nu}(q)A_\nu(q)]. \tag{14}$$

At the order of one loop

$$\pi(q) = \int \frac{\mathrm{d}^{d+1}K}{(2\pi)^{d+1}} \frac{1}{(K^2+M^2)[(K+q)^2+M^2]}$$

and satisfies $\pi(q)_{q\to 0} \neq 0$. For $\pi_{\mu\nu}(q)$ we find

$$\Pi_{\mu\nu}(q) = \int \frac{\mathrm{d}^{d+1}K}{(2\pi)^{d+1}} \left( \frac{(2K+q)_\mu (2K+q)_\nu}{(K^2+M^2)[(K+q)^2+M^2]} - \frac{2\delta_{\mu\nu}}{(K^2+M^2)} \right)$$
$$\simeq \frac{\mathrm{const}}{M^2}(q^2\delta_{\mu\nu} - q_\mu q_\nu).$$

In agreement with the gauge invariance we have $q_\mu \pi_{\mu\nu}(q) = 0$. The field $v(q)$ is massive and can be neglected. The only important term is the second one, which is identical to the action of the electromagnetic field:

$$\Gamma(A_\mu, \lambda) \simeq \Gamma(0, M^2) + \frac{N}{M^2} \int \mathrm{d}^d r \int \mathrm{d}\tau (F_{\mu\nu})^2 = \Gamma(0, M^2)$$
$$+ \frac{N}{M^2} \int \mathrm{d}^d r \int \mathrm{d}\tau [(\partial_\tau A_r)^2 + (\nabla \times A_r)^2 + (\nabla A_\tau)^2]. \tag{15}$$

Equation (15) was obtained for large $N$ and can be problematic for our case $N = 2$. It is expected on the basis of gauge invariance that, for $J < J_c$, a $1/N^2$ correction will not change the form of equation (15). The effect of the $1/N^2$ will be to renormalize $M \to M_{\mathrm{R}}$. This expansion is valid only for massive $Z$ particles $M \neq 0$. For $d = 3$ we find that the particles $Z$ have a gap $M$ for $\gamma > \gamma_c$ ($J < J_c$). The value of the gap is obtained by the solution $(\partial\Gamma/\partial\lambda)|_{\lambda=M^2} = 0$ and is given for $d = 3$ for $D = 2 + 1$ by the relation $M \sim \Lambda[1 - (1/4^2\pi\gamma_0)]$ [13]. When we rescale the action given in equation (15) such that it takes the form of the electromagnetic action, we find $A \to (M/2\sqrt{N})A$. As in electrodynamics we identify the coupling constant of $A$ with a new 'charge' $e^* \stackrel{\text{def}}{=} M/2\sqrt{N}$. (When $J \to J_c$ the gap $M$ and the charge vanish.) Therefore this description is valid only in the disordered spin phase $J < J_c$.

In equation (15) we have separated the longitudinal part from the transverse one. In the Coulomb gauge we eliminate the longitudinal component $A_\tau$. $A_\tau$ satisfies the Gauss law,

$$\nabla^2(\mathrm{i}A_\tau) = -e^*\tilde{\psi}^+(r,\tau)\sigma_3\tilde{\psi}(r,\tau)$$

with positive and negative charges given by $e^* = M/2\sqrt{N}$. Solving the Gauss equation we find the following effective Hamiltonian:

$$\begin{aligned} H = \int \mathrm{d}^2r\tilde{\psi}^+(r,\tau)\left(\frac{1}{2m^*}(\partial_r + eA^{\mathrm{EM}} + \mathrm{i}e^*\sigma_3A_r)^2\right)\tilde{\psi}(r,\tau) \\ + \int \mathrm{d}^dr\int \mathrm{d}^dr'[\tilde{\psi}^+(r,\tau)\sigma_3\tilde{\psi}(r,\tau)]\left(\frac{e^{*2}}{\nabla^2}\right)(r,r')[\tilde{\psi}^+(r',\tau)\sigma_3\tilde{\psi}(r',\tau)] \\ + \tfrac{1}{2}\int \mathrm{d}^dr[(\partial_\tau A_r)^2 + (\nabla\times A_r)^2]. \end{aligned} \tag{16}$$

The effective Hamiltonian given in equation (16) is our main result. We have an attractive Coulombic potential between the holes (with opposite spins), which have positive and negative 'charges' $e^*$ (in addition the holes have positive electromagnetic charge $e$). For $\eta = 0$ ($d = 2$) we find an attractive potential $V(r) = -e^{*2}\ln(r/a)$, and for $\eta \neq 0$ ($d = 3$) we have $V(r) = -e^{*2}/r$. The origin of this attractive Coulombic potential is the $z_1, z_2$ bosons from which the holes gain the 'charge' $\pm e^*$. The attraction is a result of exchange of a gapless gauge boson $A_\mu$ between two holes of opposite 'charges' $e^*$, $-e^*$ with the same electromagnetic charge $e$.

Equation (16) is valid for the range of parameters $t < I$, $I \geqslant J$ and $J < J_c$. This equation was obtained fwith the aid of two approximations, the $1/N$ expansion in equation (15) and the result given in equation (10). If the approximation in equation (10) is not used we have to substitute in equation (16) $\tilde{\psi}^+[\sigma_3 + \alpha(\sigma - \sigma_3)]\psi$ instead of $\tilde{\psi}\sigma_3\tilde{\psi}$ with $\alpha \ll 1$ ($\alpha$ is the $xy$ anisotropy). This will not change the basic results for small $\alpha$, which is a function of $t/I < 1$. as $t/I$ decreases, the spin-flip parameter $\alpha$ decreases and the spin-flip length $l_\alpha$ increases. Therefore we need that $l_\alpha > M^{-1}$ ($M^{-1}$ is the spin correlation length in the disordered phase). This condition $l_\alpha M > 1$ is fulfilled for $t/I < 1$, $I \geqslant J$, $J < J_c$, and the spin-flip effect is negligible.

It is important to remark that the final Hamiltonian given in equation (16) does not contain the charged bosons $z_1$, $z_2$ and therefore $M = z^+\sigma z$. They were completely integrated (see equation (13)). This is only possible when $\langle\lambda\rangle = M^2 \neq 0$ such that the magnetic correlation length (in the disordered AF phase) is much smaller than the correlation length of the gauge field $A_\mu$.

An additional comment can be made on the physics described by the action $\Gamma(A,\lambda)$ (see equation (15)). When $M^2 > M_c^2$ the gauge field $A_\mu$ has an infinite correlation length (the photon phase). In this case the correlation length $1/M$ of the charged bosons $z_1$, $z_2$

and spins $M$ is short. In the opposite limit $M^2 < M_c^2$ the 'photon' phase is destroyed and the 'charged' bosons form, at large distances, neutral spin variables $M$.

In this case the Hamiltonian (16) is not valid since the spin variables have long correlation lengths and cannot disappear from the action.

According to Polyakov [13] a critical value of $M_c^2$ exists in 3 + 1 dimensions. It is for this reason that we need the antiferromagnetic coupling $\eta$ between the plane to be non-zero (see equation (3)). The result of $\eta \neq 0$ is that the infrared fluctuations are in 3 + 1 dimensions.

## 4. The binding problem—discussions

In the remaining part of the paper we will concentrate on solving the Hamiltonian given in equation (16) at the mean-field level. Owing to the fact that $e^*$ is not small and the hole concentration is small, it makes no sense to solve the Bardeen–Cooper–Schrieffer (BCS) equation for the gap $\Delta(K)$ given by

$$\Delta(K) = \sum_{K'} \frac{e^{*2}}{(K-K')^2} \frac{\Delta(K')}{2\{[\varepsilon(K') - \mu]^2 + \Delta^2(K')\}^{1/2}}. \tag{17}$$

We look now for the solution of two holes, and check if the two holes *bind* together. The variational wavefunction for two holes is given by

$$|\varphi\rangle = \sum_{r_1 r_2} \chi(r_1 - r_2) b^+(r_1, r_2)|0\rangle$$

where $|0\rangle$ is the vacuum with no holes and

$$b^+(r_1, r_2) = \sum_{\sigma=+,-} \sigma \bar{\psi}_\sigma^+(r_1) \bar{\psi}_{-\sigma}^+(r_2)$$

is the two-hole creation operator. Performing the variation $\langle\varphi|H|\varphi\rangle$ with $\langle\varphi|\varphi\rangle = 1$ we find the following eigenvalue equation in the centre-of-mass system:

$$[-2\tilde{t}\nabla_r^2 + v(r)]\chi(r) = \lambda\chi(r) \qquad \tilde{t} \sim 1/(2m^*). \tag{18}$$

We assume an anisotropic three-dimensional system with $V(r) = -e^{*2}/r$ (this is not completely justified since the electronic dispersion in the $z$ direction is much smaller than that in the plane). For the isotropic case, we find a hydrogen-like solution with eigenvalues $\lambda_n = -e^{*2}/tn^2$ and with a binding radius (the Bohr radius) $\xi = \tilde{t}/e^{*2}$. In order to consider a realistic situation we assume the potential $V(r) = -e^{*2}/(r^2 + d^2)^{1/2}$, where $r$ is the distance between the hole in the plane and $d$ is the distance between the planes.

As a result the binding occurs between holes in adjacent planes. Such a potential has a bound-state solution. This can easily be seen if we take $V(r) = -e^{*2}/d$ for $r < d$ and $V(r) = 0$ for $r > d$ (this problem was solved in [13] p 84).

The existence of a bound-state solution for two holes leads to a picture similar to the Wannier excitons. First we obtain a bound state for two holes and assume that the pairs of holes interact via the residual interaction between the pairs. This picture is valid for the dilute limit of holes. We consider the case $\delta_{\min} < \delta < \delta_{\max}$ and $\xi \setminus 1/\sqrt{\delta} = L$ ($L$ is the average distance between the holes and $\xi \sim \tilde{t}/e^{*2}$ is the binding radius of two holes). When $\xi = L$ this picture is not valid and we have no superconductivity. The relation $\xi = L$ determines $\delta_{\max} = 1/\xi^2$.

The residual interaction between the pairs at large distances is negligible since the charge of the pair is zero. (When we consider electromagnetic effects the interaction between the pairs is $(2e)^2/R$ for $R \gg \xi$; $e^*$ is replaced by $e^* - e$ for $R \leqslant \xi$.) The effective Hamiltonian for the pairs is a boson model with a hard-core repulsive interaction, which has a superfluid transition [16] as a function of the pair concentration $\delta/2$.

We compare our results with other models that make use of spin fluctuations for superconductivity.

Using magnetic interaction, we show that in certain ranges of parameters the magnetic fluctuations are described by the fluctuations of two 'charged' boson fields $z_1$, $z_2$, which exchange a gauge field $A_\mu$ (similar to the exchange of a photon between two charged particles).

We do not use directly spin fluctuations (like other authors [1–8]) but rather their constituents. The spin fluctuation becomes the natural description when we approach the Néel phase. In this case the correlation length $1/M$ of the $z_1$, $z_2$ fields increases $M \to 0$), the gauge field $a_\mu$ loses its infinite correlation length (the photon becomes massive) and our picture becomes invalid.

For the one-band Hubbard model in the limit of small $U$, Wen, Zhang and Schrieffer [16] have used a background field similar to the one described by equation (6). However, they consider normal spin fluctuations and we show that the relevant fluctuations in our case are the 'electromagnetic' type, which lead to $1/r$ interactions.

To conclude, we have shown that gauge fields induced by antiferromagnetic fluctuations in the disordered phase lead to binding of holes. The bosonic fluid created by the whole pairs has a superfluid transition [15].

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